

Towards a global unified height system

Laura Sánchez

Vice-president of the *Global Geodetic Observing System (GGOS)* of the
International Association of Geodesy (IAG)

Technische Universität München
Deutsches Geodätisches Forschungsinstitut (DGFI-TUM)

with support and contributions from many colleagues



Bundesamt
für Eich- und
Vermessungswesen



Earth System research requires **unified geodetic reference frames** with

- an order of **accuracy higher** than the magnitude of the effects to be observed (e.g. global change);
- consistency and reliability worldwide (**the same accuracy everywhere**);
- long-term stability (**the same accuracy at any time**).

The ITRS and its realization (ITRF) provide

- geometric coordinates $(\mathbf{X}, \dot{\mathbf{X}})$ **consistent globally**;
- accuracy at **mm ... cm** level.

The **existing physical height systems** exhibit

- more than **100 realizations** worldwide;
- discrepancies of **dm ... m** (different vertical datums, different physical heights, missing standardization);
- static heights $\rightarrow \dot{H} \equiv 0$;
- imprecise combination with geometric heights $|h - H - N| \rightarrow \gg 0$;
- 1 ... 2 order of **accuracy less** than $(\mathbf{X}, \dot{\mathbf{X}})$.

→ A core objective of the **International Association of Geodesy (IAG)** is to provide an **international standard for precise determination of physical heights**.

Definition of the International Height Reference System (IHRS)

IAG Resolution No. 1, Prague, July 2015

- 1) Vertical coordinates are potential differences with respect to a conventionally fixed W_0 value:

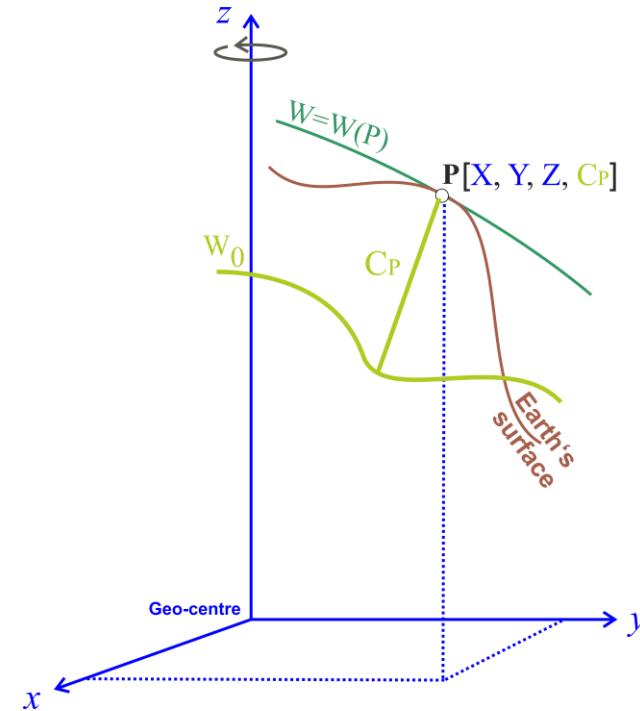
$$C_P = C(P) = W_0 - W(P) = -\Delta W(P)$$

$$W_0 = \text{const.} = 62\,636\,853.4 \text{ m}^2\text{s}^{-2}$$

- 2) The position P is given in the ITRF

$$\mathbf{X}_P (X_P, Y_P, Z_P); \text{ i.e., } W(P) = W(\mathbf{X}_P)$$

- 3) The estimation of $\mathbf{X}(P)$, $W(P)$ (or $C(P)$) includes their variation with time; i.e., $\dot{\mathbf{X}}(P)$, $\dot{W}(P)$ (or $\dot{C}(P)$).
- 4) Coordinates are given in mean-tide system / mean (zero) crust.
- 5) The unit of length is the meter and the unit of time is the second (SI).



- For the IAG resolutions, see Drewes et al. (2016), *The Geodesist's Handbook 2016*, J Geod, <https://doi.org/10.1007/s00190-016-0948-z>
- Ihde et al. (2017), *Definition and proposed realization of the International Height Reference System (IHRS)*. Surv Geophy 38(3), 549-570, <https://doi.org/10.1007/s10712-017-9409-3>
- Sánchez et al. (2016), *A conventional value for the geoid reference potential W_0* , J Geod, 90(9): 815-835, <https://doi.org/10.1007/s00190-016-0913-x>,

Realisation of the IHRS

A reference frame realises a reference system in two ways:

- physically, by a **solid materialisation of points** (or observing instruments),
- mathematically, by the **determination of coordinates** referring to that reference system. The coordinates of the points are computed from the measurements following the definition of the reference system.

The implementation of the IHRS mainly requires:

- 1) A global **reference network** for the IHRS realisation: the International Height Reference Frame (IHRF)
- 2) **The determination of reference IHRF coordinates ($W, \dot{W}, \mathbf{X}, \dot{\mathbf{X}}$)** at the reference stations
- 3) Clear **standards, conventions and procedures** to ensure consistency between the definition (IHRS) and the realisation (IHRF)
- 4) **Operational and organisational infrastructures** (reference stations, data centres, analysis centres, combination centres, product centres, etc.) to guarantee maintenance and availability of the IHRF.

Criteria for the IHRF reference network configuration

1) Hierarchy:

- A **global network** → worldwide distribution, including
- A **core network** → to ensure sustainability and long term stability
- **Regional and national densifications** → local accessibility

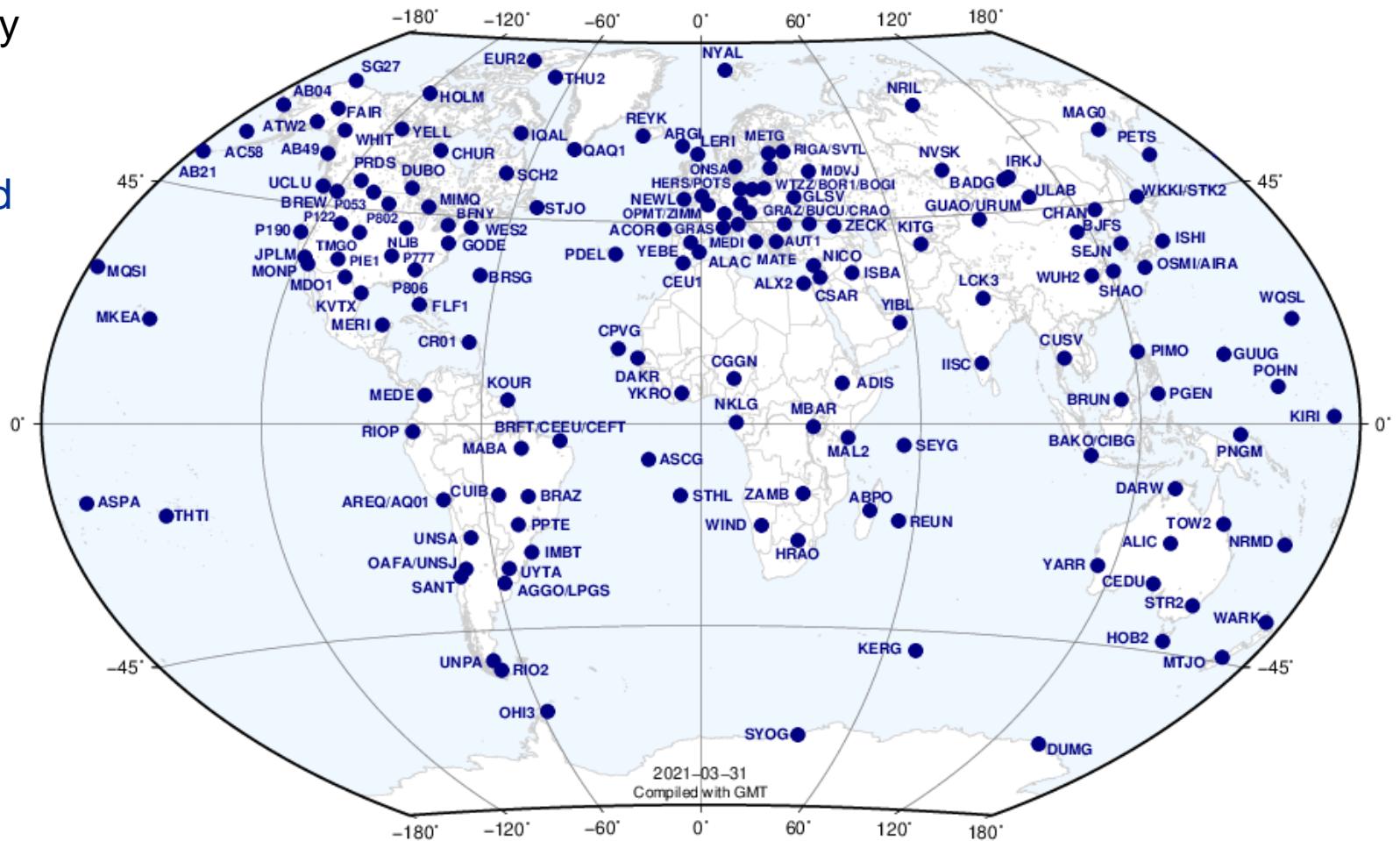
2) Collocated with:

- fundamental **geodetic observatories** → connection between \mathbf{X} , \mathbf{W} , \mathbf{g} and time realisation (reference clocks);
- **continuously operating reference stations** → to detect deformations of the reference frame (preference for ITRF and regional reference stations, like SIRGAS, EPN, APREF, etc.);
- **reference tide gauges and national vertical networks** → to facilitate the vertical datum unification;
- reference stations of the new **International Gravity Reference Frame - IGRF** → to integrate the gravity and physical height reference frames.

3) Main requirement: availability of surface gravity data around the IHRF reference stations for high-resolution gravity field modelling (i.e., precise estimation of \mathbf{W}).

First proposal for the IHRF reference network (~170 stations)

- Station selection coordinated by the **GGOS-FA Unified Height System** in agreement with the **GGOS Bureau of Networks and Observations**, the **Bureau Gravimétrique International** (absolute gravity stations), as well as with the **IAG regional sub-commissions for reference frames and gravity field modelling**.
- A **living network**: new stations and decommission of stations.
- To be **extended** by regional/national densifications.



Basic considerations on the ITRS/IHRF coordinates

- 1) The ITRS/IHRF is based on the combination of
 - a geometric component given by the coordinate vector **X** in the ITRS/ITRF and
 - a physical component given by the determination of potential values **W** at **X**.
- 2) The determination of **X** follows the [IERS Conventions](#), a similar documentation for the determination of the potential values should be achieved.
- 3) To be in agreement with the reliability of the ITRF, the expected accuracy of **W** is
 - Positions: $\approx \pm 3 \times 10^{-2} \text{ m}^2\text{s}^{-2}$ (about [3 mm](#))
 - Velocities: $\approx \pm 3 \times 10^{-3} \text{ m}^2\text{s}^{-2}/\text{a}$ (about [0.3 mm/a](#))
- 4) For the moment, our goal is $\pm 1 \times 10^{-1} \text{ m}^2\text{s}^{-2}$ (about [1 cm](#))
- 5) The ITRS/IHRF coordinates include the determination/modelling of time variations. For the moment, we consider [static coordinates only](#).

Approaches for the determination of IHRF potential coordinates

- 1) Global gravity models of high resolution (GGM-HR), including topography-based synthetic gravity signals (like the model XGM2019e (Zingerle et al., 2019), or the combination of GGMs of degree > 2156 with topography potential models like Earth2014 (Rexer et al., 2016), ERTM2160 (Hirt et al., 2014), etc.)
- 2) Precise regional gravity field modelling (methods for the geoid/quasi-geoid determination)

$$W(P) = U(P) + T(P) \quad [\text{m}^2\text{s}^{-2}]$$

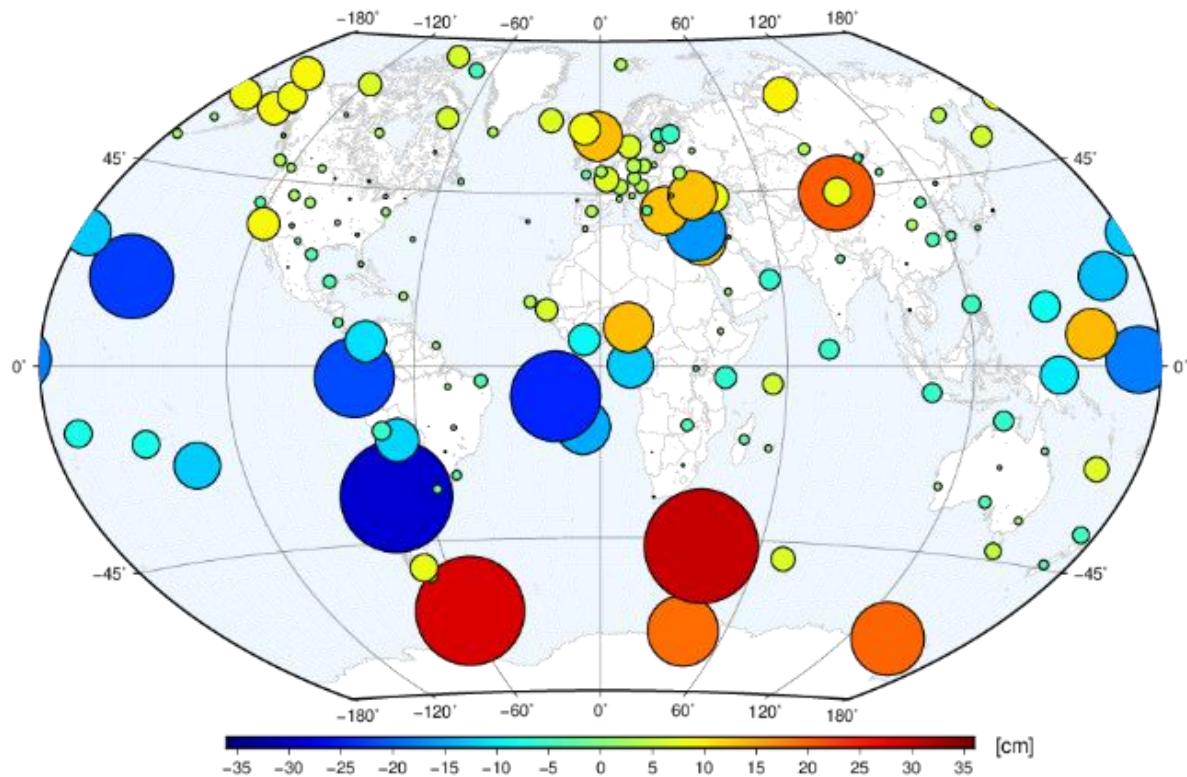
Quasi-geoid $W(P) = U(P) + \zeta(P) \cdot \gamma_Q + \Delta W_0 \quad [\text{m}^2\text{s}^{-2}] \rightarrow W(P) = W_0 - (h(P) - \zeta(P)) \cdot \bar{\gamma}_{QQ_0} \quad [\text{m}^2\text{s}^{-2}]$

Geoid $W(P) = W_0 - (h(P) - N(P)) \cdot \bar{g}(P) \quad [\text{m}^2\text{s}^{-2}]$ with
 $\bar{g}(P) = g(P) + 0.424 \times 10^{-6} \cdot (h(P) - N(P)) + TC(P) \quad [\text{ms}^{-2}]$

- 3) Vertical datum unification of the local height systems into the IHRF (details discussed by Sánchez and Sideris (2017),
<https://doi.org/10.1093/gji/ggx025>, not further considered here)

$$W(P) = (W_0^{local} + \delta W) - C_P^{local} \quad \text{with} \quad \delta W = W_0^{IHRF} - W_0^{local}$$

IHRF potential coordinates based on global gravity models



Mean: 0.5 cm, STD: ± 8.2 cm, Min: -30.5 cm, Max: 31.0 cm

See Sánchez et al. (2021), *Strategy for the realisation of the IHRF*,
J Geod 95, 33 (2021). <https://doi.org/10.1007/s00190-021-01481-0>

- Comparison of normal heights inferred from **XGM2019e (degree 5540)** with the mean value of the normal heights obtained from **EIGEN-6C4 (degree 2190; Förste et al., 2015)**, **GECO (degree 2190; Gilardoni et al., 2016)**, and **SGG-UGM-1 (degree 2159; Liang et al., 2018)**.
- These differences are mainly attributable to the contribution of **surface gravity data posterior to EGM2008 (Pavlis et al., 2012)** and the gravity synthetic effects inferred from the **Earth2014 topography model**.
- To evaluate the reliability of these values, independent data (e.g., levelling+gravimetry) are needed.
- New and better surface gravity data distribution and quality (as in preparation for the EGM2020) will strongly improve the GGM-based estimates.

IHRF potential coordinates based on regional gravity field modelling

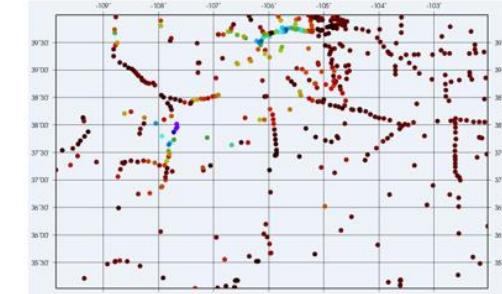
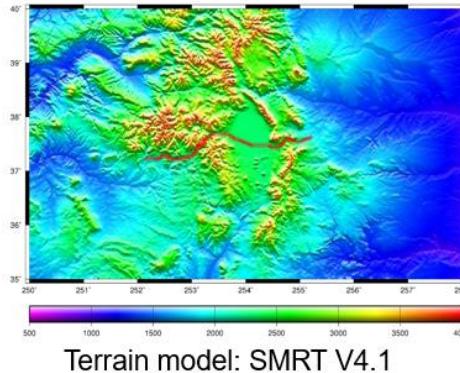
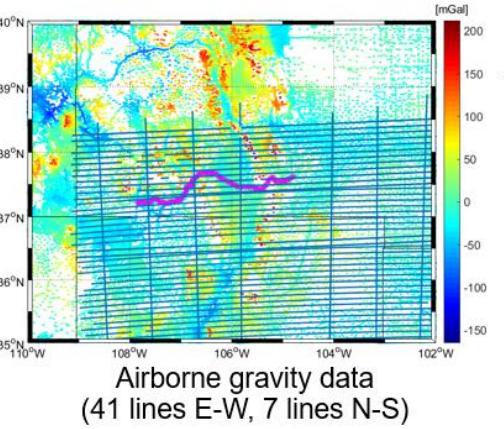
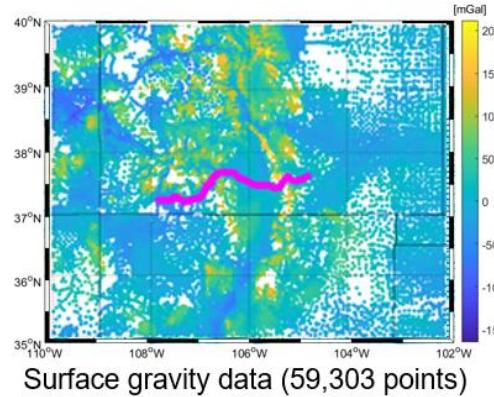
- GGMs based on SLR, GRACE and GOCE are **very precise** ($\pm 1 \dots \pm 2$ cm @ 100 km)
- Mean omission error globally: $\approx \pm 45$ cm
- Goal is to **reduce these ± 45 cm to ± 1 cm** by solving the Geodetic Boundary Value Problem (GBVP) with the combination of a GGM + surface gravity data + topography effects

$$W_P = U_P + T_P \quad T_P = T_{P,\text{satellite-only}} + T_{P,\text{residual}} + T_{P,\text{terrain}}$$

- The determination of T_P demands a series of approximations, which influence the results; i.e., **different methodologies produce different potential values**
- A “centralised” computation (like in the ITRF) is quite complicated due to the restricted accessibility to surface gravity data. So, regional/national experts have to be involved in the determination of the potential coordinates in their regions/countries
- A “standard” computation procedure may be not appropriate as
 - different data availability and different data quality exist around the world
 - regions with different characteristics require particular approaches (e.g. modification of kernel functions, size of integration caps, geophysical reductions like GIA, etc.).

Comparison of computation methods: Colorado experiment

Objective: to compute **geoid**, **quasi-geoid** and **potential values** using exactly the same input data, a set of **basic standards**, and the **own methodologies** (software) of colleagues involved in the gravity field modelling.

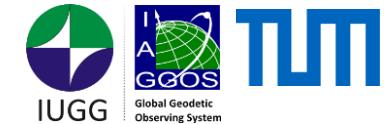


NGS historical GPS/levelling (509 points)

See Wang et al. (2021) *Colorado geoid computation experiment: overview and summary*. J Geod, 95(12), <https://doi.org/10.1007/s00190-021-01567-9>.

- Initiated in July 2017
- Data provided by US NGS
- Standards prepared by L Sánchez, J Ågren, J Huang, YM Wang, R Forsberg
- Three computations (two iterations) finished in June 2019
- Fourteen (final) contributing solutions
- Special Issue on “**Reference Systems in Physical Geodesy**” of the Journal of Geodesy with computation methods and comparison of geoid and quasi-geoid models, see <https://link.springer.com/journal/volumesAndIssues/190?tabName=topicalCollections>

Colorado experiment: contributing solutions



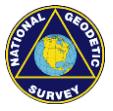
Istanbul Technical University,
Istanbul, [Turkey](#)



Faculty of Engineering, Minia
University, [Egypt](#)



Department of Geodesy and
Surveying, Aristotle University of
Thessaloniki, Thessaloniki, [Greece](#)



National Geodetic Survey, [USA](#)



Natural Resources Canada,
[Canada](#)



Lantmäteriet, Swedish mapping,
cadastral and land registration
authority, [Sweden](#)



School of Earth and Planetary
Sciences and The Institute for
Geoscience Research, Curtin
University, [Australia](#)



Deutsches Geodätisches
Forschungsinstitut, Technische
Universität München, [Germany](#)



Ingenieurinstitut für
Astronomische und Physikalische
Geodäsie, Technische Universität
München, [Germany](#)



Chinese Academy of Surveying
and Mapping, [China](#)



Politecnico di Milano, [Italy](#)



Faculty of Geodesy, University of
Zagreb, [Croatia](#) - Research
Institute of Geodesy, Topography
and Cartography, [Czech Republic](#)



National Space Institute,
Technical University of Denmark,
[Denmark](#)



Geography and Crustal Dynamics
Research Center, Geospatial
Information Authority of Japan,
[Japan](#)

Colorado experiment: summary of approaches and models

- GGMs: GOCO05s, XGM2016, XGM2018, xGEOD17B, EIGEN-6C4
- Topographic effects based on SRTM V4.1, EARTH2014, ERTM2160

Solutions based on the quasi-geoid computation using FFT integration and a Wong-Gore modification of the integral kernel



A

Solutions based on the quasi-geoid computation with a least-squares modification of Stokes' formula with additive corrections (LSMSA)



B

Solutions based on the quasi-geoid computation using spherical radial basis functions (9) and least-squares collocation (10, 11)



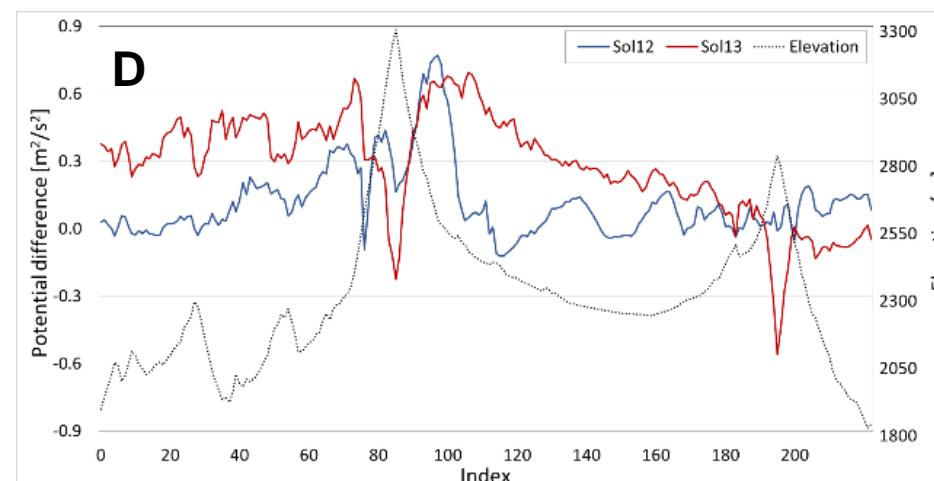
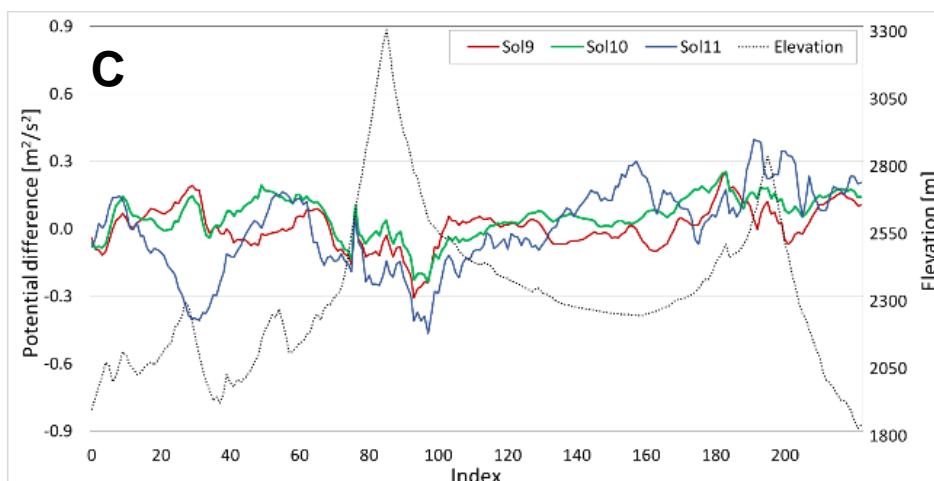
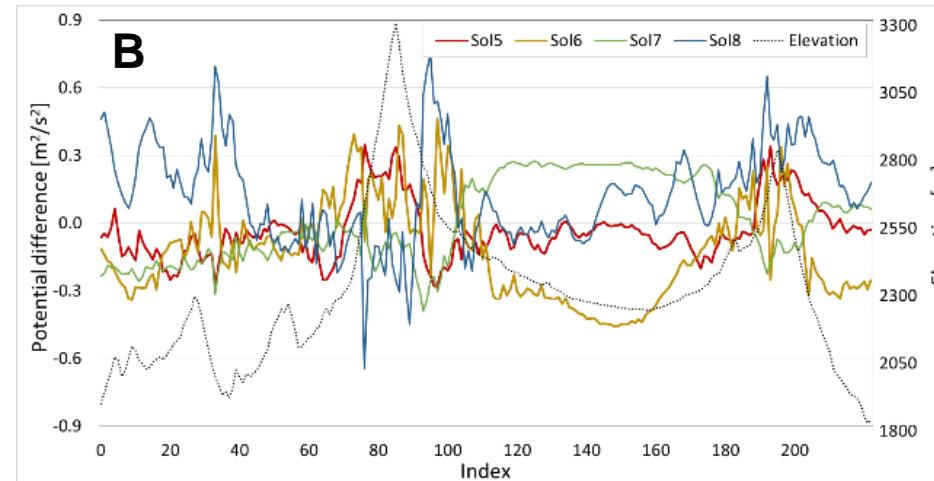
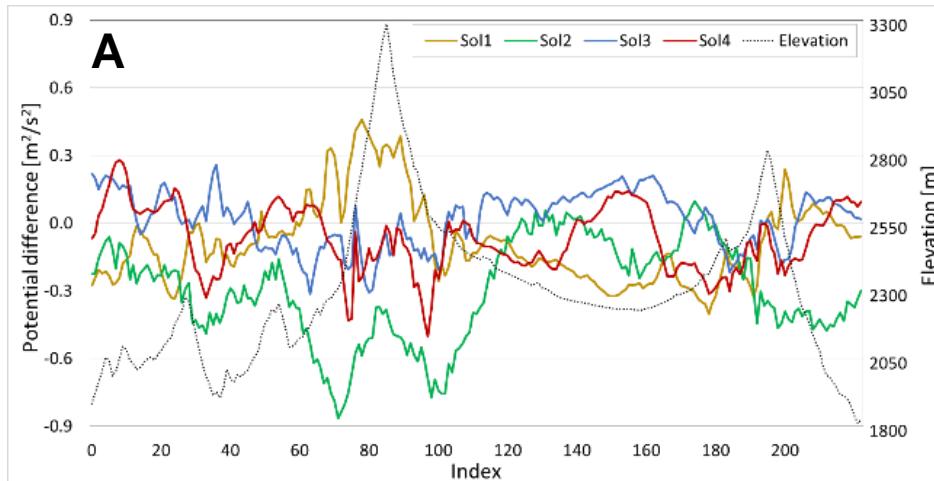
C

Solutions based on the geoid computation using the Helmert-Stokes (H-S) scheme and then converted to the quasi-geoid



D

Colorado experiment: comparison of potential values wrt mean value (model – mean)

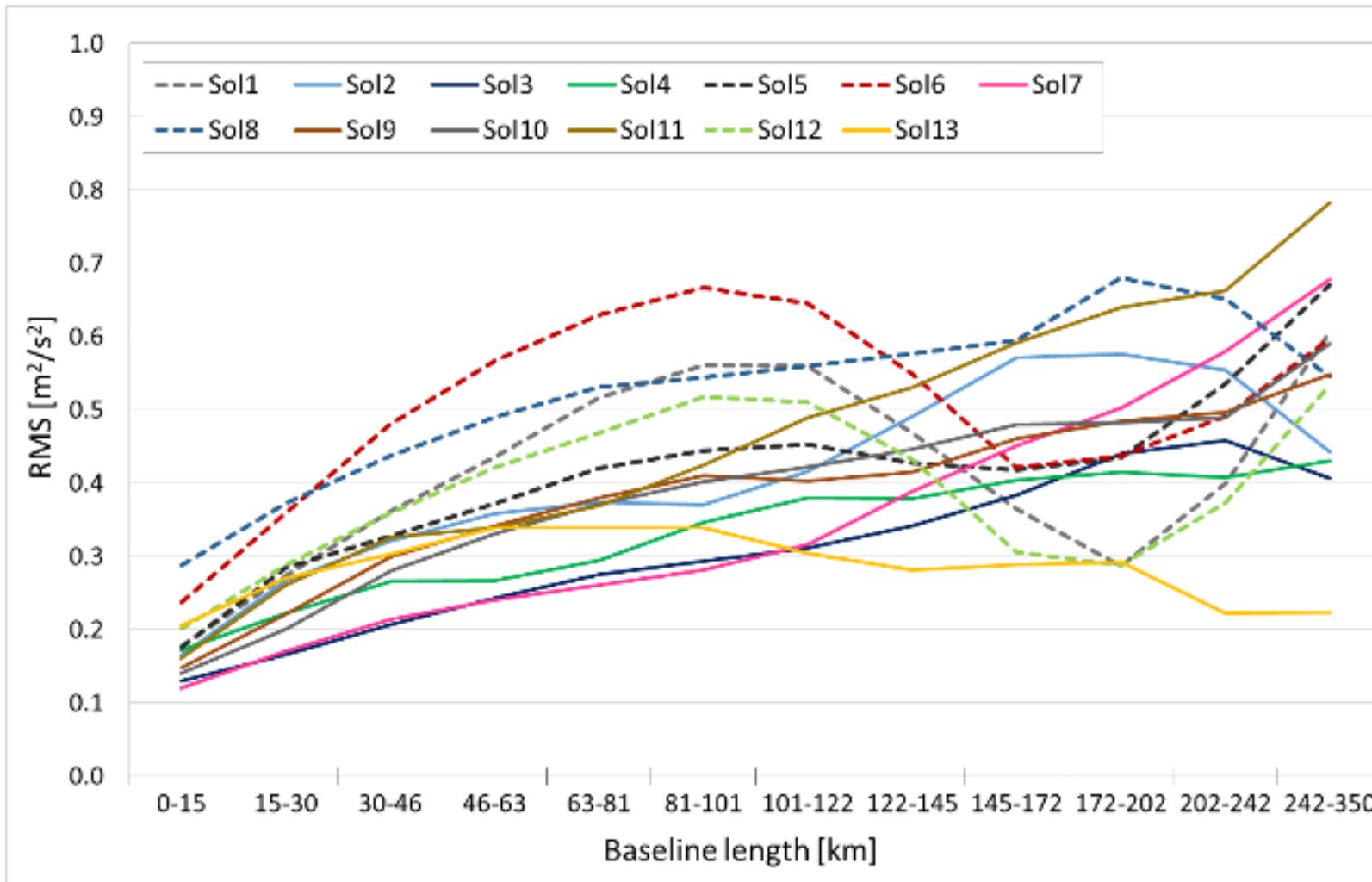


- Agreement within $\pm 0.09 \text{ m}^2\text{s}^{-2}$ ($\pm 0.9 \text{ cm}$) and $\pm 0.23 \text{ m}^2\text{s}^{-2}$ ($\pm 2.3 \text{ cm}$) in terms of STD wrt mean value.
- The overall differences range from $-0.86 \text{ m}^2\text{s}^{-2}$ (-8.8 cm) to $+0.77 \text{ m}^2\text{s}^{-2}$ ($+7.9 \text{ cm}$).
- Discrepancies present a strong correlation with the topography.

See Sánchez et al. (2021), *Strategy for the realisation of the IHRS*, J Geod 95, 33 (2021).

<https://doi.org/10.1007/s00190-021-01481-0>

Colorado experiment: comparison of potential values with levelling (+ gravimetry)



The RMS values of the ΔC_{ij} differences for each interval indicates the consistency between the model-based and levelling-based potential values as a function of the distance.

See Sánchez et al. (2021), *Strategy for the realisation of the ITRS*, J Geod 95, 33 (2021).
<https://doi.org/10.1007/s00190-021-01481-0>

Learnings from the Colorado experiment

The determination of potential values may be classified in three main scenarios:

a) **Regions without (or with very few) surface gravity data,**

- The only option to determine potential values is the use of GGM-HRs
- Expected mean accuracy values around the $\pm 4.0 \text{ m}^2\text{s}^{-2}$ ($\pm 40.0 \text{ cm}$) level or even worse in regions with strong topography gradients
- It could be improved for instance to the $\pm 1.0 \text{ m}^2\text{s}^{-2}$ ($\pm 10.0 \text{ cm}$) level if new and better surface gravity data are included in the GGMs.
- To avoid multiple potential values provided by different GGM-HRs at the same point, it is necessary to select one GGM-HR as reference model.

b) **Regions with some surface gravity data, but with poor data coverage or unknown data quality,**

- The reliability of the existing (quasi-)geoid models is poor
- Additional gravity surveys around the IHRF stations to increase the accuracy of the geopotential numbers computed at those specific stations.

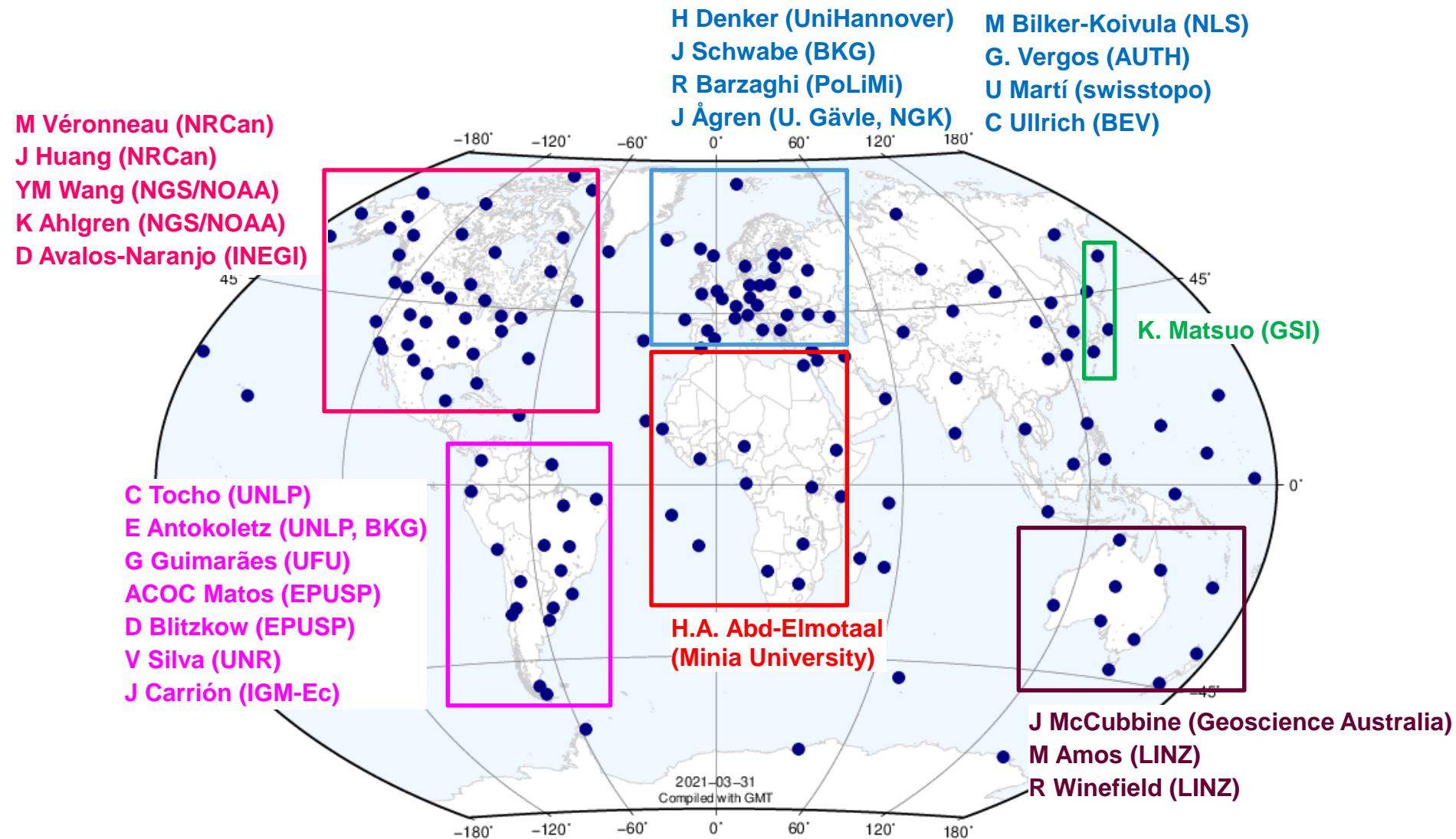
c) **Regions with good surface gravity data coverage and quality.**

- Potential values may be inferred from precise (quasi-)geoid regional models

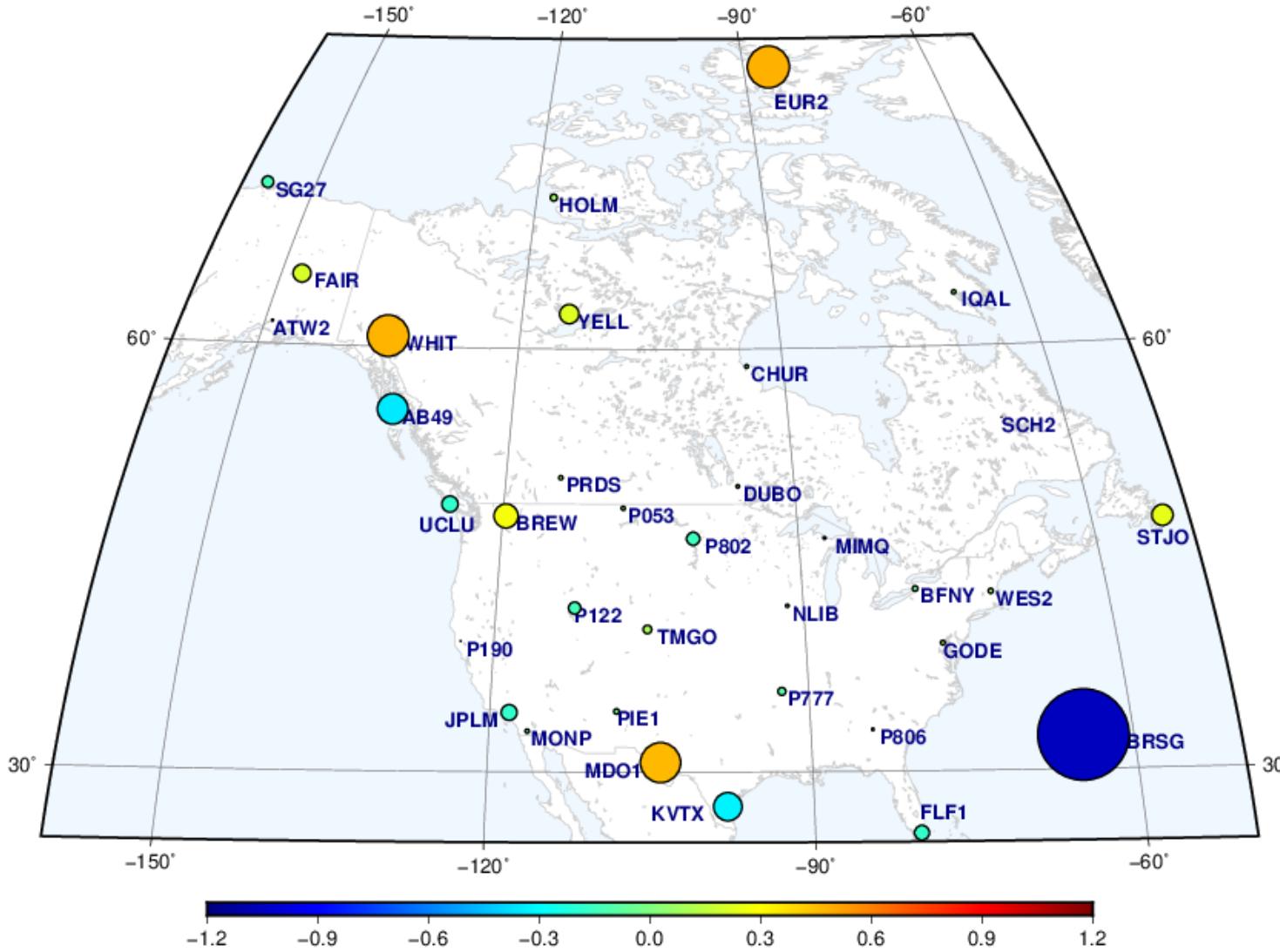
On going activities: Computation of a first solution for the IHRF

- 1) Recovering of IHRF potential values from national and regional (quasi-)geoid models with the support of IAG regional sub-commissions, national/regional experts in geoid modelling, and the geoid repository of the International Service for the Geoid (ISG)
- 2) Computation of potential values using the latest **GGM + topography** signals from Earth2014 and ERTM2160
- 3) Comparison of (1) and (2)
 - to decide on the GGM to be used in regions with no (quasi-)geoid model available and
 - to evaluate the reliability of regional models with poor gravity data distribution

On going activities: Computation of a first solution for the IHRF



On going activities: Computation of a first solution for the IHRF



Differences between the potential values inferred from the Canadian geoid model PCGG20_21A and the US quasi-geoid model xG20B
(thanks to M Véronneau, J Huang, YM Wang and K Ahlgren):

Mean: $-0.01 \text{ m}^2\text{s}^{-2}$
STD: $0.26 \text{ m}^2\text{s}^{-2}$
Min.: $-1.05 \text{ m}^2\text{s}^{-2}$
Max.: $0.48 \text{ m}^2\text{s}^{-2}$

On going activities: Computation of a first solution for the IHRF

EGG2016	GCG2016	Difference
18032.74	18032.81	0.07
15946.68	15946.73	0.05
15381.77	15381.78	0.02
1019.01	1018.97	-0.04
16690.78	16690.73	-0.05
6070.26	6070.18	-0.08

IHRF geopotential numbers inferred from the European quasi-geoid model EGG2016 and the German quasi-geoid model GCG2026
(thanks to H Denker and J Schwabe)

Differences (@ 6 points)

Mean: $-0.006 \text{ m}^2\text{s}^{-2}$

STD: $0.050 \text{ m}^2\text{s}^{-2}$

Min.: $-0.080 \text{ m}^2\text{s}^{-2}$

Max.: $0.065 \text{ m}^2\text{s}^{-2}$

On going activities: Computation of a first solution for the IHRF

Canada (11 stations):
Model PCGG20_21A
Mean accuracy $0.35 \text{ m}^2\text{s}^{-2}$

Europe (40 stations):
Model EGG2016
Mean accuracy $0.50 \text{ m}^2\text{s}^{-2}$

US main territory (30 stations):
Model NAPGD2022 - xG20B
Mean accuracy $0.45 \text{ m}^2\text{s}^{-2}$

In progress:

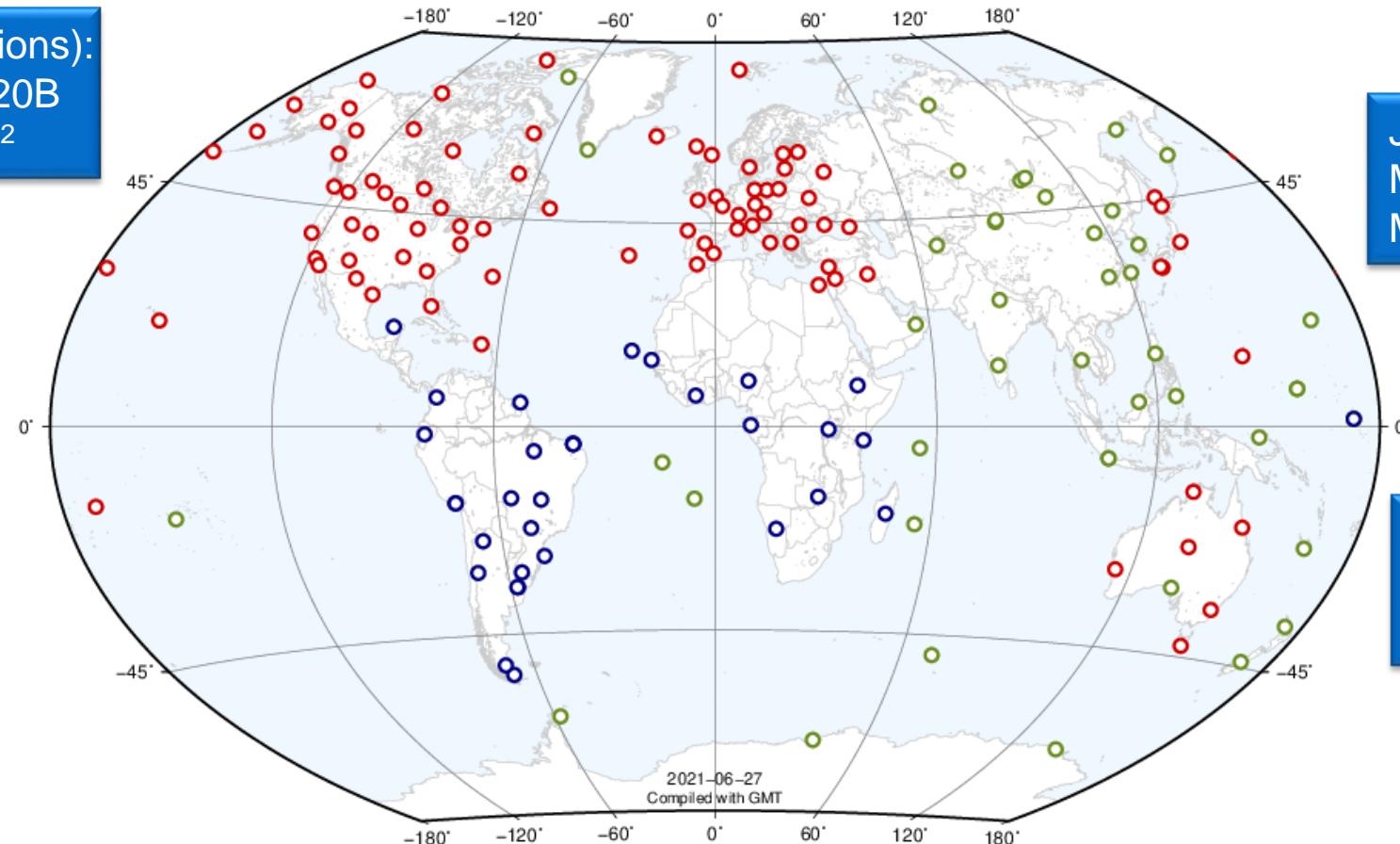
South America:
Comparison of regional
and national solutions

Africa: computation of
potential values

Asia and Oceania:
Inventory of ISG geoid
repository or selection
of GGM

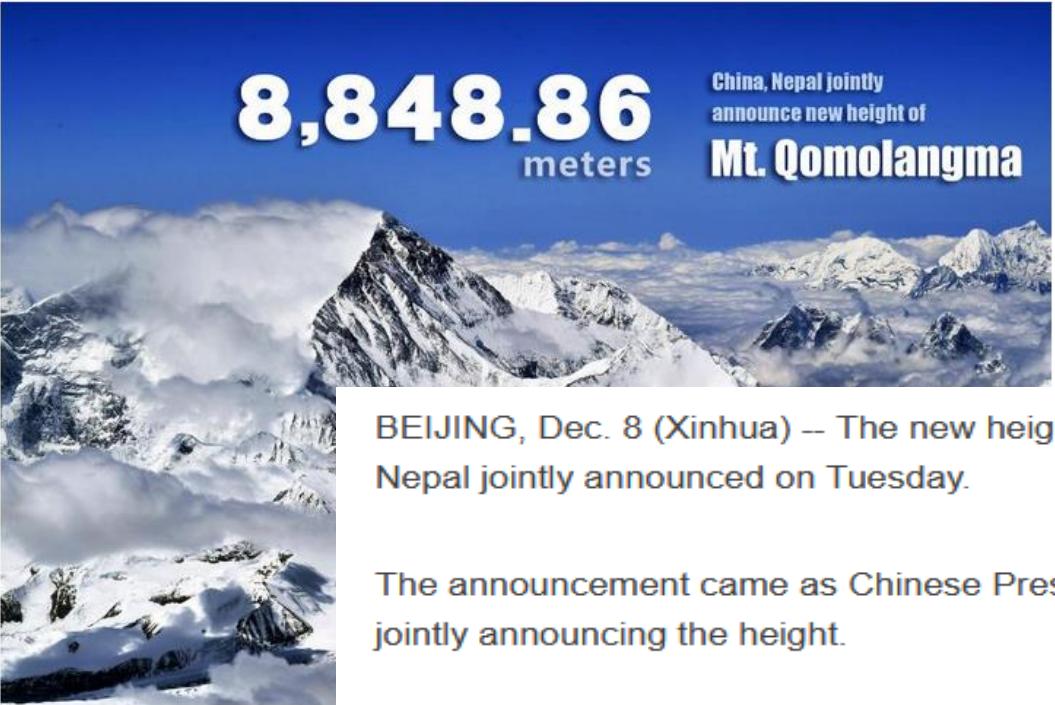
Japan (5 stations):
Model JGEOD2019
Mean accuracy $0.57 \text{ m}^2\text{s}^{-2}$

Australia (6 stations):
Model AGQG2017
Mean accuracy $0.62 \text{ m}^2\text{s}^{-2}$



Closing remarks

- 1) The determination of **the gravity potential** $W(P) = U(P) + T(P)$ is the core element for the establishment of the IHRS/IHRF.
- 2) The comparison of different computation strategies (the Colorado experiment) proves that we can reach an agreement of about $0.2 \text{ m}^2\text{s}^{-2}$ ($\sim 2 \text{ cm}$). However, this depends on the **availability of surface gravity data**. When no gravity data is available, the uncertainty may reach $10 \text{ m}^2\text{s}^{-2}$ ($\sim 1 \text{ m}$).
- 3) The establishment and maintenance of the IHRF is only possible in the frame of a **strong international collaboration** like that provided by the IAG and its Scientific Services.
- 4) The installation of an ‘IHRS/IHRF element’ within the International Gravity Field Service (IGFS) is planned to ensure the **maintenance and availability of the IHRF**:
 - Regular updates of the IHRF to take account for new stations;
 - coordinate changes with time $\dot{\mathbf{X}}, \dot{W}$;
 - improvements in the estimation of \mathbf{X} and W (more observations, better standards, better models, better computation algorithms, etc.).



8,848.86 meters -- China, Nepal jointly announce new height of Mt. Qomolangma

Source: Xinhua | 2020-12-08 18:03:44 | Editor: huaxia

BEIJING, Dec. 8 (Xinhua) -- The new height of Mount Qomolangma, the world's highest peak, is 8,848.86 meters, China and Nepal jointly announced on Tuesday.

The announcement came as Chinese President Xi Jinping and his Nepali counterpart Bidya Devi Bhandari exchanged letters jointly announcing the height.

Xi said in his letter that China and Nepal reached consensus last year on the joint announcement of the new height of the peak.

For more than a year, the two countries' survey teams have overcome all kinds of difficulties, solidly carried out their work, and finally reached a conclusion on the snow-covered height based on the International Height Reference System, he said.

Calling Mount Qomolangma "an important symbol of the China-Nepal traditional friendship," Xi said it is agreed by both countries as the boundary peak and the "Peak of China-Nepal Friendship."

Xi said the joint announcement of the new height of Mount Qomolangma with his Nepali counterpart is of great significance in carrying forward the undertakings of the predecessors to the future, and showcasing the high level of the continuous development of China-Nepal relations.